■機械系数学　演習問題 No.1　微分積分の復習　(担当: 谷戸)

1. 次の関数を微分せよ.

(1) $y = x^2 + \dfrac{1}{x^2} + \sqrt{x}\ \ (x > 0)$

(2) $y = e^{-3x}$

(3) $y = 2\sin 5x - 3\cos 5x$

(4) $y = \log(-x)\ \ (x < 0)$

(5) $y = e^{2x}\cos 7x$

2. 次の不定積分を求めよ.

(1) $\displaystyle\int (x^3 - 6x + 2)\,dx$

(2) $\displaystyle\int \frac{1}{x - 10}\,dx$

(3) $\displaystyle\int \frac{2x}{x^2 + 3}\,dx$

(4) $\displaystyle\int e^{4x}\,dx$

(5) $\displaystyle\int x\sin x\,dx$

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■機械系数学　演習問題 No.1　微分積分の復習　(担当: 谷戸)【解答】

1. (1) $y' = \left(x^2 + x^{-2} + x^{\frac{1}{2}}\right)' = 2x - 2x^{-3} + \frac{1}{2}x^{-\frac{1}{2}} = 2x - \dfrac{2}{x^3} + \dfrac{1}{2\sqrt{x}}$

   (2) $y' = \left(e^{-3x}\right)' = e^{-3x} \cdot (-3x)' = -3e^{-3x}$

   (3) $y' = 2 \cdot (\sin 5x)' - 3 \cdot (\cos 5x)' = 2 \cdot (5\cos 5x) - 3 \cdot (-5\sin 5x) = 10\cos 5x + 15\sin 5x$

   (4) $y' = \{\log(-x)\}' = \dfrac{1}{-x} \cdot (-x)' = \dfrac{1}{-x} \cdot (-1) = \dfrac{1}{x}$

   (5) 積の微分法より

$$\begin{aligned} y' &= (e^{2x})' \cdot \cos 7x + e^{2x} \cdot (\cos 7x)' \\ &= (2e^{2x}) \cdot \cos 7x + e^{2x} \cdot (-7\sin 7x) \\ &= 2e^{2x}\cos 7x - 7e^{2x}\sin 7x \end{aligned}$$

2. 以下, C は積分定数.

   (1)

$$\int (x^3 - 6x + 2)\,dx = \frac{1}{4}x^4 - 3x^2 + 2x + C$$

   (2) 置換積分法を用いる. $t = x - 10$ とおくと, $\frac{dt}{dx} = (x-10)' = 1$ なので, $dt = dx$. よって,

$$\begin{aligned} \int \frac{1}{x-10}\,dx &= \int \frac{1}{t}\,dt \\ &= \log|t| + C \\ &= \log|x-10| + C \end{aligned}$$

   (3) 置換積分法を用いる. $t = x^2 + 3$ とおくと, $\frac{dt}{dx} = (x^2+3)' = 2x$ なので, $dt = 2x\,dx$. よって,

$$\begin{aligned} \int \frac{2x}{x^2+3}\,dx &= \int \frac{1}{x^2+3} \cdot 2x\,dx \\ &= \int \frac{1}{t}\,dt \\ &= \log|t| + C \\ &= \log|x^2+3| + C \\ &= \log(x^2+3) + C \quad (\text{注: } x^2+3 > 0 \text{ より}) \end{aligned}$$

   (4) 置換積分法を用いる. $t = 4x$ とおくと, $\frac{dt}{dx} = 4$ なので, $dx = \frac{1}{4}\,dt$. よって,

$$\begin{aligned} \int e^{4x}\,dx &= \int e^t \cdot \frac{1}{4}\,dt \\ &= \frac{1}{4}\int e^t\,dt \\ &= \frac{1}{4}e^t + C \\ &= \frac{1}{4}e^{4x} + C \end{aligned}$$

   (5) 部分積分法を用いる.

$$\begin{aligned} \int x\sin x\,dx &= x \cdot (-\cos x) - \int 1 \cdot (-\cos x)\,dx \\ &= -x\cos x + \int \cos x\,dx \\ &= -x\cos x + \sin x + C \end{aligned}$$